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The equation of motion of an interstellar Bussard ramjet

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Abstract

An interstellar Bussard ramjet is a spaceship using the protons of the interstellar medium in a fusion engine to produce thrust. The velocity of an ideal ramjet (100% efficient hydrogen fusion engine) could, in theory, approach the speed of light in a time of the order of years, or less. The relativistic effects for such a spaceship would be very large. The parametric equations, in terms of the ramjet's speed, for the position of the ramjet in the inertial frame of the interstellar medium, the time in this frame, and the proper time indicated by the clocks on board the spaceship, are presented in an analytical form. The nonrelativistic and ultrarelativistic limits are studied. The possibility of using interstellar antimatter as a source of energy is also discussed.

1. Introduction

In 1960 Robert Bussard published the first technical paper on the interstellar fusion ramjet [1]. During its motion through space, this type of spacecraft collects interstellar ions (mostly protons) with a magnetic scoop (or ramscoop). The charged particles are funnelled into a fusion reactor capable of fusing protons to obtain helium. A possible reaction is the proton–proton chain ($4\text{p} \to {}^4\text{He} + 2\text{e}^+ + 2\nu$) burning four protons into one helium nucleus, plus two positrons and two electron neutrinos, and releasing about 0.7% of the initial rest mass as energy [2]. Accelerated reaction products are exhausted from the spacecraft's rear, which produces thrust. Bussard estimated the ramjet's performance in an ionized, high density region of interstellar medium. He found that an ideal ramjet (100% efficient hydrogen fusion engine), with a mass of 1000 t and an effective intake area of 10^4 km^2 moving through a gas of protons with density of 10^9 m^{-3}, could accelerate at 1 g, starting with a very low initial speed. The ramjet velocity could then approach the speed of light within a year.

Unfortunately, more realistic studies [3, 4] show that the ramjet performance could be strongly limited by the constraints on structural strength of materials, and by mass and radiation losses. Moreover, the reaction cross section of the proton–proton fusion reaction is orders of magnitude lower than that of the deuterium–tritium fusion reaction, which might be used to produce energy on Earth in the (near) future. This is due to the fact that the proton–proton reaction speed is controlled by a weak process transforming a proton into a neutron. Finally, the interstellar medium in the neighbourhood of the Sun is expected to be composed mainly of neutral hydrogen atoms with a density lower than 10^6 m^{-3}.

However, the ramjet concept is potentially too valuable to be simply discarded despite its tremendous technical difficulties. Some researchers have suggested alternatives to the initially proposed proton–proton fusion ramjet [5, 6]: the catalytic ramjet (the use of catalyzed fusion reaction with a high rate), the RAIR (the use of nuclear fuel carried by the ship), etc. We discuss here the possibility of using interstellar antimatter as propellant.

In this paper, we will consider only the ideal ramjet, for a one-dimensional rectilinear flight in free space, and for which thrust and acceleration vectors are parallel. Partial results about the corresponding equations of motion can be found in several works [1, 3, 4], but we will show here that analytical formulae can be obtained for the position of the ramjet in the inertial frame of the interstellar medium, the time in this frame, and the proper time indicated by the clocks on board the spaceship. Parametric equations of these three quantities are given in terms of the ramjet's speed. The solutions have very simple nonrelativistic and ultrarelativistic limits.

It is not very probable that an ideal ramjet could be built one day, even in the far future. This work can be considered as an advanced exercise (from the point of view of calculus) in special relativity. The framework could be very attractive for undergraduate students, especially those interested in (interstellar) space travel. The basic equations are simple outcomes of momentum and energy conservation. Even if the solutions are not evident to obtain, they have interesting properties to explore.

2. General equations

In the following, all calculations are performed in the frame of the interstellar medium, considered as an inertial frame. The interstellar medium contains protons at rest with a mass density ρ. A Bussard ramjet of constant rest mass M moves at speed $v = \beta c$ through this interstellar medium. The effective intake area of the ramscoop is denoted by A. A fraction ϵ of the reaction mass is converted into exhaust kinetic energy in the 100% efficient hydrogen fusion reactor. This means that if a mass dm of protons at rest is scooped up from the interstellar medium, an energy $\epsilon\,\mathrm{d}m\,c^2$ is released and totally converted into ordered motion of the exhausted material with a rest mass $(1-\epsilon)\,\mathrm{d}m$. We assume that no fraction of energy is lost, for instance, in the form of thermal radiation (no radiation loss), and we assume that no fraction of the collected interstellar gas is lost during the operation of the engine (no mass loss).

During the time interval dt, a mass dm of protons, at rest, is scooped up. The ramjet speed is then increased by the quantity $c\,\mathrm{d}\beta$, thanks to the ejection of a mass $(1-\epsilon)\,\mathrm{d}m$ of protons with a speed wc. The conservation of momentum implies that

$$M\gamma(\beta)\beta c = M\gamma(\beta+\mathrm{d}\beta)(\beta+\mathrm{d}\beta)c + (1-\epsilon)\,\mathrm{d}m\,\gamma(w)wc, \qquad (1)$$

where $\gamma(x) = 1/\sqrt{1-x^2}$. Let us note that $w < 0$ with $\mathrm{d}\beta > 0$. The conservation of energy leads to

$$M\gamma(\beta)c^2 + \mathrm{d}m\,c^2 = M\gamma(\beta+\mathrm{d}\beta)c^2 + (1-\epsilon)\,\mathrm{d}m\,\gamma(w)c^2. \qquad (2)$$

The collected mass is given by

$$dm = A\rho\beta c\, dt. \tag{3}$$

Taking into account that $\gamma(\beta + d\beta) = \gamma(\beta) + \gamma(\beta)^3\beta\, d\beta$, we obtain the following sets of equations:

$$M\gamma(\beta)^3\, d\beta + (1-\epsilon)A\rho\beta\, dt\, \gamma(w)wc = 0, \tag{4}$$

$$M\gamma(\beta)^3\beta\, d\beta + (1-\epsilon)A\rho\beta c\, dt\, \gamma(w) = A\rho\beta c\, dt. \tag{5}$$

3. Acceleration

The division of two equations (4) and (5) by dt/c gives

$$M\gamma(\beta)^3\varphi + (1-\epsilon)A\rho\beta\gamma(w)wc^2 = 0, \tag{6}$$

$$M\gamma(\beta)^3\beta\varphi + (1-\epsilon)A\rho\beta c^2\gamma(w) = A\rho\beta c^2, \tag{7}$$

where $\varphi = dv/dt$ is the acceleration measured in the rest frame of the interstellar medium. The elimination of the reduced speed w of the exhausted reaction mass by the relation $\gamma(w)^2(1-w^2) = 1$ gives the second degree equation in φ

$$\left(\frac{M\gamma^2\varphi}{A\rho c}\right)^2 + 2c\beta^2\gamma\frac{M\gamma^2\varphi}{A\rho c} - \beta^2c^2\epsilon(2-\epsilon) = 0, \tag{8}$$

in which we adopt the notation $\gamma = \gamma(\beta)$. The physical solution ($\varphi > 0$) is

$$\frac{M\gamma^2\varphi}{A\rho c} = -c\beta^2\gamma + \sqrt{(c\beta^2\gamma)^2 + \beta^2c^2\epsilon(2-\epsilon)}. \tag{9}$$

We can now define characteristic acceleration φ_*, time t_* and length x_* by the following relations:

$$\varphi_* = \frac{A\rho c^2}{M}, \qquad t_* = \frac{c}{\varphi_*}, \qquad x_* = \frac{c^2}{\varphi_*}. \tag{10}$$

It is also natural to introduce the new parameter

$$\epsilon' = \epsilon(2-\epsilon), \tag{11}$$

ϵ is only 0.0071 for the most energetic known fusion reaction [1]. So, for all fusion reactions, we have $\epsilon' \approx 2\epsilon$.

With the notation defined above, we find

$$\frac{\varphi}{\varphi_*} = \beta\sqrt{1-\beta^2}\left[\sqrt{\beta^2 + \epsilon'(1-\beta^2)} - \beta\right]. \tag{12}$$

This equation gives the acceleration of the ramjet as a function of its speed in the inertial frame of the interstellar medium (see figure 1). Obviously, $\varphi = 0$ when $\epsilon = 0$, but acceleration also vanishes if $\beta = 0$. A initial boosting is thus necessary for the ramjet. This is due to the fact that the reaction mass reaches the reactor, thanks to the speed of the ramjet. In theory, a very low speed is sufficient to start the ramjet. In practice, a fusion reactor could probably not operate correctly without a sufficient intake. Nevertheless the ramjet could accelerate with an initial speed as low as 10 km s^{-1} [1]. Such a speed could be reached with usual chemical rockets.

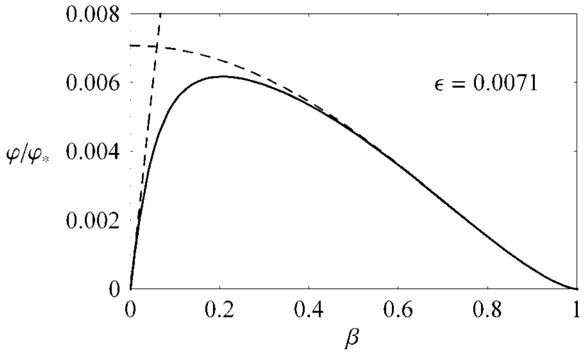

Figure 1. Reduced acceleration φ/φ_* of the ramjet in the inertial frame of the interstellar medium as a function of its reduced speed $\beta = v/c$, for the parameter $\epsilon = 0.0071$ [1]. Dashed lines show the approximations for nonrelativistic and ultrarelativistic speeds.

Equation (12) possesses two interesting limits. When $\beta \ll 1$, the nonrelativistic limit, equation (12) reduces to

$$\varphi = \varphi_* \beta \sqrt{\epsilon'} = \frac{\sqrt{\epsilon'}}{t_*} v, \tag{13}$$

that is to say the equation for an exponential motion with a characteristic time $t_c = t_*/\sqrt{\epsilon'}$ (see appendix A). When $\beta \approx 1$, the ultrarelativistic limit, a first-order expansion can be performed for the square root of equation (12) and we obtain

$$\varphi = \varphi_* \frac{\epsilon'}{2} (1-\beta^2)^{3/2}, \tag{14}$$

that is to say the equation of motion for a uniformly accelerated spaceship with acceleration $\varphi_c = \varphi_* \epsilon'/2$ (see appendix B).

The acceleration $\varphi(0)$ measured on board is given by $\varphi/(1-\beta^2)^{3/2}$ [7]. This is a monotonic increasing function of β

$$\frac{\varphi(0)}{\varphi_*} = \frac{\beta}{1-\beta^2} \left[\sqrt{\beta^2 + \epsilon'(1-\beta^2)} - \beta \right]. \tag{15}$$

As expected from the ultrarelativistic limit, we have

$$\lim_{\beta \to 1} \varphi(0) = \varphi_* \frac{\epsilon'}{2}. \tag{16}$$

In the following, we will present the parametric equations of motion of the ramjet as a function of its reduced speed β. We will assume that, at a time $t = 0$ in the inertial frame, the position of the ramjet is $x = 0$ and its reduced speed $\beta_0 \neq 0$. Moreover, the clocks on board the ramjet indicate a proper time $\tau = 0$.

4. Time

The solution of equation (12) is given by the following integral ($\varphi/\varphi_* = t_* \mathrm{d}\beta/\mathrm{d}t$):

$$\frac{1}{t_*} \int_0^t \mathrm{d}t = \int_{\beta_0}^{\beta} \frac{\mathrm{d}\beta}{\beta\sqrt{1-\beta^2}\left[\sqrt{\beta^2 + \epsilon'(1-\beta^2)} - \beta\right]}. \tag{17}$$

An analytical formula for the rhs integral has been found with the help of Mathematica. After some manipulations to obtain a simpler form, the equation is

$$\frac{t}{t_*} = \frac{1}{\sqrt{\epsilon'}} \ln\left[\frac{\sqrt{\epsilon'(1-\beta_0^2)} + \sqrt{\beta_0^2 + \epsilon'(1-\beta_0^2)}}{\sqrt{\epsilon'(1-\beta^2)} + \sqrt{\beta^2 + \epsilon'(1-\beta^2)}} \frac{\beta}{\beta_0}\right] + \frac{1}{\epsilon'}\left[\frac{\beta + \sqrt{\beta^2 + \epsilon'(1-\beta^2)}}{\sqrt{1-\beta^2}} - \frac{\beta_0 + \sqrt{\beta_0^2 + \epsilon'(1-\beta_0^2)}}{\sqrt{1-\beta_0^2}}\right]. \tag{18}$$

It is clear from this equation that β_0 cannot vanish.

In the nonrelativistic limit ($\beta \ll 1$ and $\beta_0 \ll 1$), the second term of the rhs of equation (18) vanishes, and the equation becomes

$$\frac{t}{t_*} \approx \frac{1}{\sqrt{\epsilon'}} \ln\left[\frac{\beta}{\beta_0}\right]. \tag{19}$$

This yields the exponential motion with the characteristic time $t_c = t_*/\sqrt{\epsilon'}$ (see equation (A.2)). In the ultrarelativistic regime ($\beta \approx 1$ and $\beta_0 \approx 1$), the first term of the rhs of equation (18) vanishes, and the equation becomes

$$t \approx \frac{t_*}{\epsilon'}\left[\frac{2\beta}{\sqrt{1-\beta^2}} - \frac{2\beta_0}{\sqrt{1-\beta_0^2}}\right]. \tag{20}$$

This yields the uniformly accelerated motion with acceleration $\varphi_c = \varphi_* \epsilon'/2$ (see equation (B.2)). The elapsed time in the inertial frame (18) is the sum of two contributions: the first term of the rhs is large only for small speed, while the second term is large only at relativistic speed.

5. Proper time

Using the well-known relation between the time and the proper time $\mathrm{d}\tau = \mathrm{d}t/\gamma$ [7], equation (12) simplifies to give

$$t_* \frac{\mathrm{d}\beta}{\mathrm{d}\tau} = \beta\left[\sqrt{\beta^2 + \epsilon'(1-\beta^2)} - \beta\right]. \tag{21}$$

The analytical solution of this equation reads

$$\frac{\tau}{t_*} = \frac{1}{\sqrt{\epsilon'}} \ln\left[\frac{\sqrt{\beta_0^2 + \epsilon'(1-\beta_0^2)} + \sqrt{\epsilon'}}{\sqrt{\beta^2 + \epsilon'(1-\beta^2)} + \sqrt{\epsilon'}} \frac{\beta}{\beta_0}\right] + \frac{1}{\epsilon'} \ln\left[\frac{\sqrt{\beta^2 + \epsilon'(1-\beta^2)} + 1}{\sqrt{\beta_0^2 + \epsilon'(1-\beta_0^2)} + 1} \frac{1-\beta_0}{1-\beta}\right]. \tag{22}$$

It is again interesting to study the two natural limits of this formula. In the nonrelativistic limit, the second term of the rhs of equation (22) vanishes, and the equation becomes identical to formula (19), since $t \approx \tau$ when $\beta \ll 1$. In the ultrarelativistic regime, the first term of the rhs of equation (22) vanishes, and the equation becomes

$$\frac{\tau}{t_*} \approx \frac{1}{\epsilon'} \ln\left[\frac{1+\beta}{1+\beta_0} \frac{1-\beta_0}{1-\beta}\right] = \frac{2}{\epsilon'}(\operatorname{arctanh}\beta - \operatorname{arctanh}\beta_0). \tag{23}$$

This yields the uniformly accelerated motion with acceleration $\varphi_c = \varphi_* \epsilon'/2$ (see equation (B.4)). The elapsed proper time on board the ramjet is also the sum of two contributions: the first term of the rhs of equation (22) is large only for low speeds, while the second term is large only at relativistic speed.

6. Distance

To find the distance travelled by the ramjet in the interstellar medium, we first divide the set of equations (4) and (5) by dx, which gives

$$M\gamma(\beta)^3\beta' + (1-\epsilon)A\rho\gamma(w)w = 0, \tag{24}$$

$$M\gamma(\beta)^3\beta\beta' + (1-\epsilon)A\rho\gamma(w) = A\rho, \tag{25}$$

where $\beta' = \mathrm{d}\beta/\mathrm{d}x$. Eliminating the reduced speed w, we find

$$\left(\frac{M\gamma^2\beta'}{A\rho}\right)^2 + 2\beta\gamma\frac{M\gamma^2\beta'}{A\rho} - \epsilon' = 0. \tag{26}$$

The physical solution ($\beta' > 0$) is

$$\frac{\mathrm{d}\beta}{\mathrm{d}x} = \frac{1}{x_*}\sqrt{1-\beta^2}\left[\sqrt{\beta^2+\epsilon'(1-\beta^2)} - \beta\right]. \tag{27}$$

Again, this differential equation can be integrated, and the solution has the following form:

$$\frac{x}{x_*} = X(\beta) - X(\beta_0)$$

with

$$X(\beta) = \frac{1}{\epsilon'}\left[\frac{\beta\sqrt{\beta^2+\epsilon'(1-\beta^2)}+1}{\sqrt{1-\beta^2}}\right] + \frac{1}{\sqrt{\epsilon'}}\left[F\left(\arcsin\beta, \frac{\epsilon'-1}{\epsilon'}\right) - E\left(\arcsin\beta, \frac{\epsilon'-1}{\epsilon'}\right)\right], \tag{28}$$

where F and E are respectively the elliptic integrals of the first and the second kind [8]. The distance x is also the sum of two contributions, but we have numerically verified that the contribution of the elliptic integrals in $X(\beta)$ does not exceed 15%, whatever the value of ϵ'. So we will neglect this contribution in the study of the limits of equation (28).

For a small ramjet speed, equation (28) reduces to

$$\frac{x}{x_*} \approx \frac{1}{\sqrt{\epsilon'}}(\beta - \beta_0). \tag{29}$$

This yields the exponential motion with the characteristic time $t_c = t_*/\sqrt{\epsilon'}$ (see equation (A.3)). For reduced speeds β and β_0 close to unity, equation (28) becomes

$$\frac{x}{x_*} \approx \frac{1}{\epsilon'}\left[\frac{2}{\sqrt{1-\beta^2}} - \frac{2}{\sqrt{1-\beta_0^2}}\right]. \tag{30}$$

This yields the uniformly accelerated motion with acceleration $\varphi_c = \varphi_*\epsilon'/2$ (see equation (B.6)).

7. Antimatter ramjet

The fraction of mass ϵ converted into energy is less than 1% in a fusion process. Larger fractions are possible only by the use of matter–antimatter reactions. Actually, a large cloud of antimatter (essentially positrons) has been detected in the galactic core [9]. A ramjet moving through this region could collect the antimatter to produce exhaust kinetic energy. As the

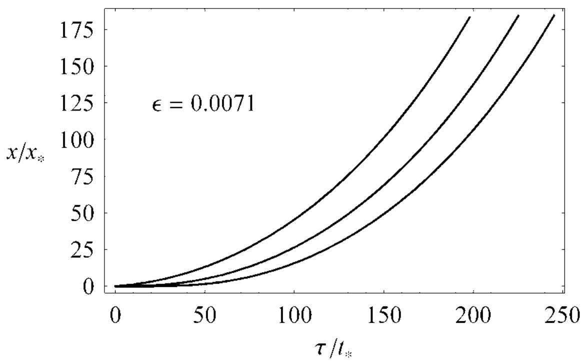

Figure 2. Reduced distance x/x_* travelled by the ramjet in the inertial frame of the interstellar medium as a function of its reduced proper time τ/t_*, for the parameter $\epsilon = 0.0071$ [1]. The three curves, from bottom to top, correspond respectively to the values of initial reduced speed $\beta_0 = 0.001, 0.01$ and 0.1.

antimatter density is expected to be lower than the matter density, the ramjet would suffer no serious damage due to direct interactions with positrons.

If the amounts of matter and antimatter collected are the same, the mass reaction can be totally converted into pure energy. This means that $\epsilon = \epsilon' = 1$ and only gamma photons are exhausted. We will study this limiting case, even if such an extreme situation seems technically out of reach. This requires the presence of perfect gamma reflectors as in the case of the pure photon rocket [5]. Moreover, the density of interstellar antimatter is probably too small (even in the galactic core) to be used efficiently as the only source of power. The momentum and energy conservation equations (1) and (2) must be modified because $w = 1$ when only photons are produced. If the released energy $\mathrm{d}E$ is used completely for the thrust, the term $(1-\epsilon)\,\mathrm{d}m\gamma(w)wc$ in equation (1) must be replaced by $-\mathrm{d}E/c$ and the term $(1-\epsilon)\,\mathrm{d}m\gamma(w)c^2$ in equation (2) must be replaced by $\mathrm{d}E$. After some simple calculations, we obtain exactly equation (12) with $\epsilon = 1$

$$\left.\frac{\varphi}{\varphi_*}\right|_{\epsilon=1} = \beta(1-\beta)\sqrt{1-\beta^2}. \tag{31}$$

A detailed calculation shows that expressions (18), (22) and (28) are still relevant when $\epsilon = 1$. We have then

$$\left.\frac{t}{t_*}\right|_{\epsilon=1} = \sqrt{\frac{1+\beta}{1-\beta}} - \sqrt{\frac{1+\beta_0}{1-\beta_0}} + \ln\left[\frac{\sqrt{1-\beta_0^2}+1}{\sqrt{1-\beta^2}+1}\frac{\beta}{\beta_0}\right], \tag{32}$$

$$\left.\frac{\tau}{t_*}\right|_{\epsilon=1} = \ln\left[\frac{1-\beta_0}{1-\beta}\frac{\beta}{\beta_0}\right], \tag{33}$$

$$\left.\frac{x}{x_*}\right|_{\epsilon=1} = \sqrt{\frac{1+\beta}{1-\beta}} - \sqrt{\frac{1+\beta_0}{1-\beta_0}}. \tag{34}$$

8. Summary

Formulae (18), (22) and (28) form the complete set of parametric equations of motion for the ideal Bussard ramjet, as a function of its speed. It is then easy to compute, for instance, the distance travelled by the ramjet in the interstellar medium as a function of the proper time indicated by the on-board clocks (see figure 2). The acceleration of the ramjet in the interstellar medium is given by equation (12), and the on-board acceleration by equation (15). The use of interstellar antimatter is also considered.

Mass loss and radiation loss are assumed here to be negligible. If it is not the case, it can be shown that the speed of the ramjet reaches a maximum value [3]. The equations of motion are then more complicated, but analytical approximate solutions can be found in the asymptotic regime.

Let us mention that an observer moving through the vacuum with a constant proper acceleration a would experience a thermal flux of radiation with a temperature $T = \hbar a/(2\pi kc)$. This is called the Unruh effect [10]. For a ramjet with accelerations bearable for human passengers, this temperature is negligible.

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Appendix A. Exponential motion

In this paper, a motion is called ‘exponential’ if the equation of motion is given by

$$\varphi = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{v}{t_c}, \tag{A.1}$$

where t_c is a characteristic time. The integration of this equation, with the condition that $v = v_0$ at $t = 0$, yields immediately

$$t = t_c \ln \frac{v}{v_0} \qquad \text{or} \qquad v = v_0 \exp(t/t_c). \tag{A.2}$$

It is clear that the motion is only possible if $v_0 \neq 0$. The time to double the speed is equal to $t_c \ln 2$. If we assume that $x = 0$ at $t = 0$, a second integration yields

$$x = t_c(v - v_0). \tag{A.3}$$

Appendix B. Uniformly accelerated motion

The equation of motion for a spaceship with a constant acceleration φ_c in its proper frame is [7]

$$\varphi = c\frac{\mathrm{d}\beta}{\mathrm{d}t} = \varphi_c(1 - \beta^2)^{3/2}. \tag{B.1}$$

Assuming that $v = v_0$ at $t = 0$, the integration of this equation gives

$$t = \frac{c}{\varphi_c}(\beta\gamma - \beta_0\gamma_0), \tag{B.2}$$

where $\gamma_0 = \gamma(v_0)$. Inserting $\mathrm{d}\tau = \mathrm{d}t/\gamma$ in equation (B.1), one finds

$$\frac{\mathrm{d}\beta}{\mathrm{d}\tau} = \frac{\varphi_c}{c}(1 - \beta^2). \tag{B.3}$$

If we assume that the clock of the spaceship and those of the inertial frame are synchronized such that $\tau = 0$ when $t = 0$, the integration of the last equation gives

$$\tau = \frac{c}{\varphi_c}(\operatorname{arctanh}\beta - \operatorname{arctanh}\beta_0). \tag{B.4}$$

Equation (B.1) can be recast into the form

$$\frac{\mathrm{d}x}{\mathrm{d}\beta} = \frac{c^2}{\varphi_c}\frac{\beta}{(1-\beta^2)^{3/2}}. \tag{B.5}$$

With the condition $x = 0$ at $t = 0$, a new integration yields

$$x = \frac{c^2}{\varphi_c}(\gamma - \gamma_0). \tag{B.6}$$

If $v_0 = 0$, a speed close to c can be reached after a time (t or τ) around the characteristic time c/φ_c. A distance around c^2/φ_c is then travelled.

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